Efficient and Accurate Phase Unwrapping Algorithms for Noisy Images within Fringe Patterns

高雑音干渉縞に対する高効率高精度位相連結アルゴリズム

Samia HESHMAT (Ph.D.)

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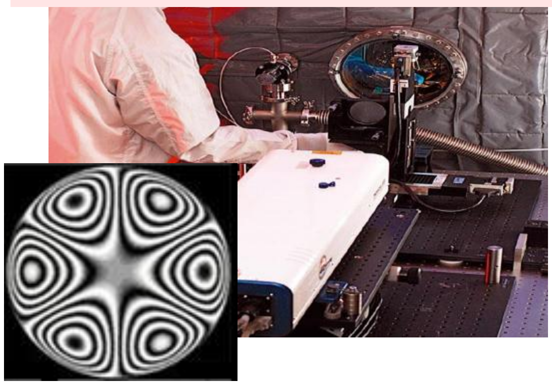
1 INTRODUCTION

- Introduction
- Fringe analysis techniques
- Research Objectives

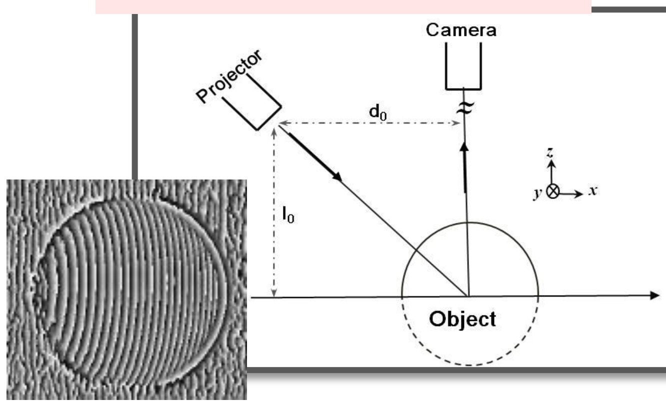
Introduction

- Optical measurement techniques such as interferometry and fringe projection Profilometry have become crucial tools in many areas of science and engineering.
- These techniques have the features of **non-contact characteristics and highly accurate measurement capability**

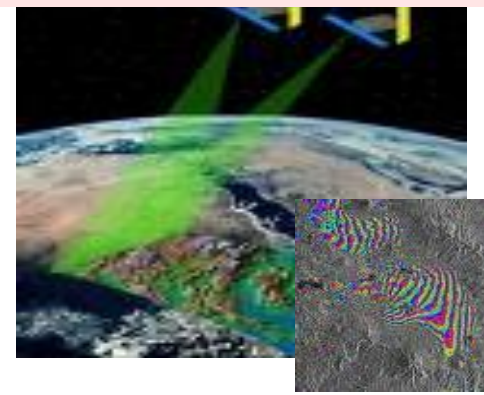
➢ Optical Interferometer



➢ FT Profilometry



➢ Synthetic Aperture Radar

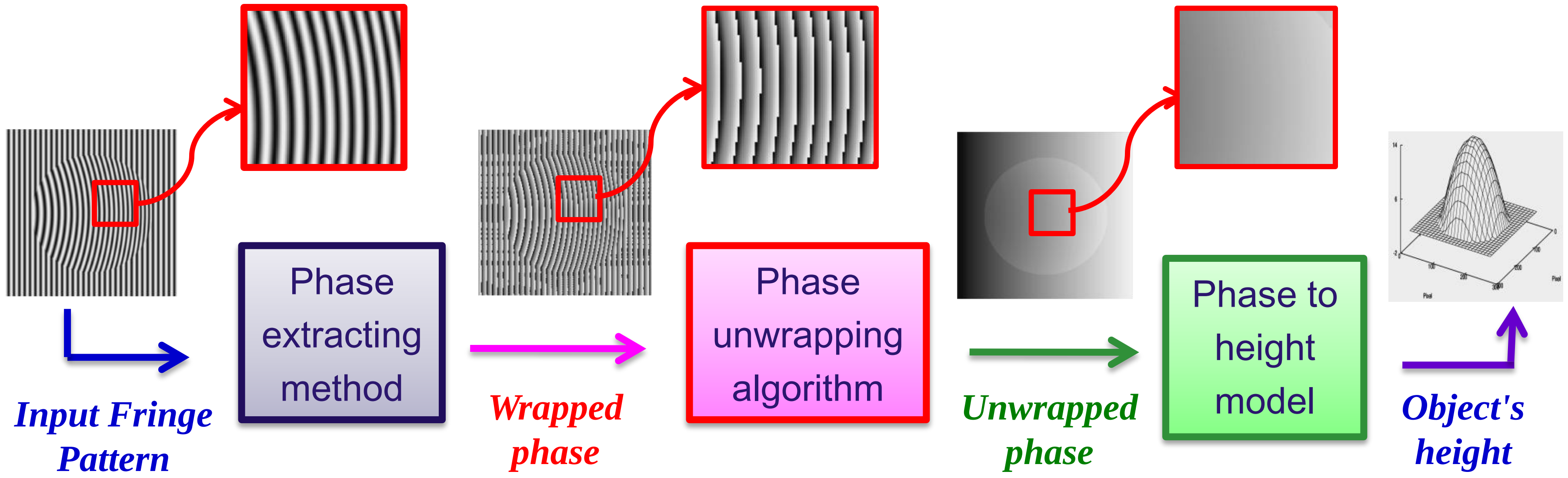


Most optical methods require the processing of a fringe pattern to obtain meaningful information

Fringe analysis techniques

◆ **Fringe analysis techniques** are used to demodulate fringe patterns.

are divided into two main processing stages:



◆ **Phase extraction stage** - extract the phase information from the fringe pattern - produces wrapped phases which their values are ranging from $-\pi$ to π

◆ **Phase unwrapping stage** - get back the continuous form of the phases at each pixel.

Fringe analysis techniques

$$I(x) = I_0 \cos(\varphi(x))$$
$$= \frac{I_0}{2}\left(e^{i\varphi} + e^{-i\varphi}\right)$$

$$\varphi = \tan^{-1}\left(\frac{Im\, I_1}{Re\, I_1}\right) + 2\pi n$$

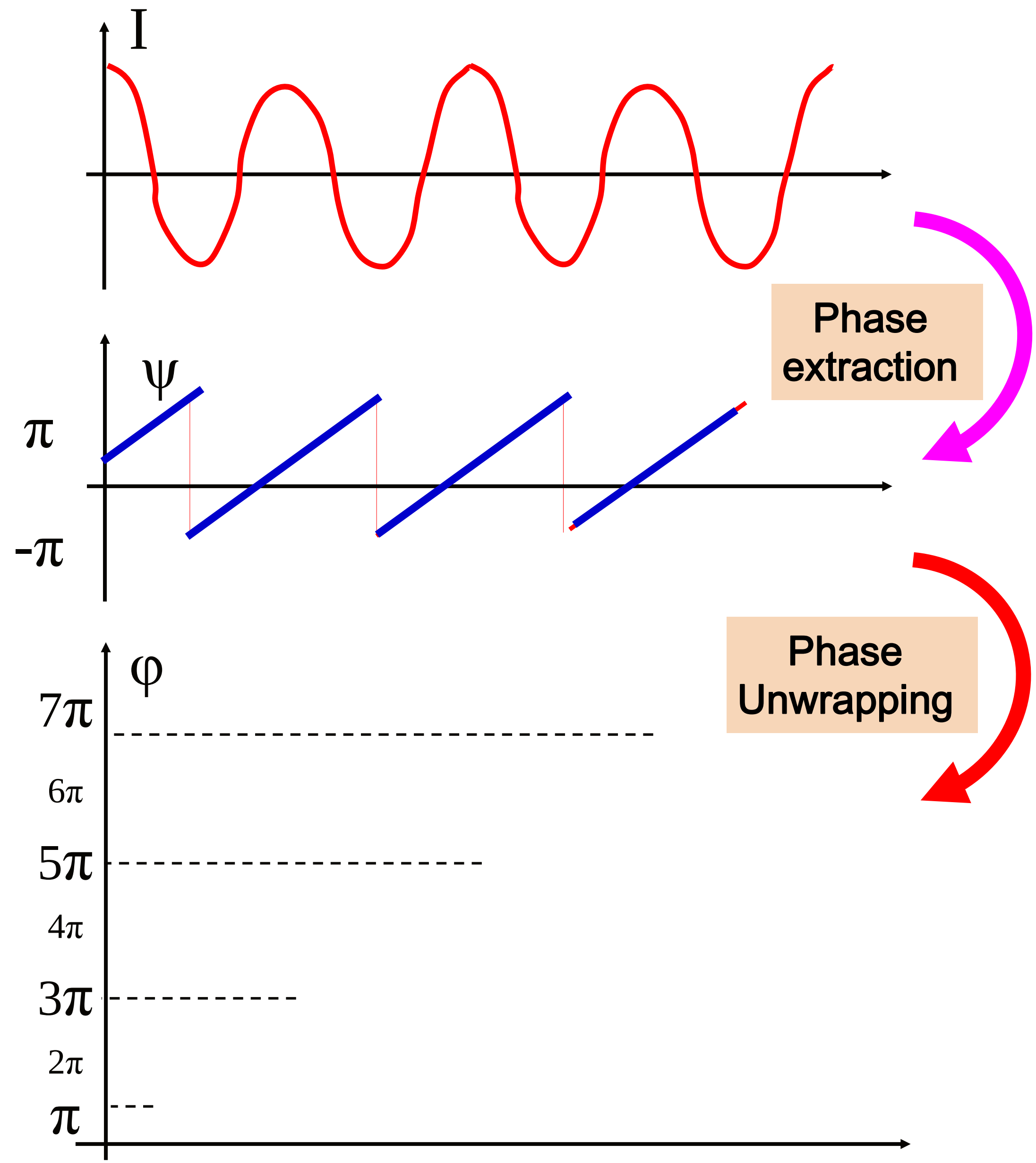
$$\psi = \tan^{-1}\left(\frac{Im\, I_1}{Re\, I_1}\right)$$

Measured phase (wrapped phase) is calculated by using the mathematical arctangent function

Phase Unwrapping:
The process of removing 2π jumps in the phase data.

$$\varphi = \psi + 2\pi n$$

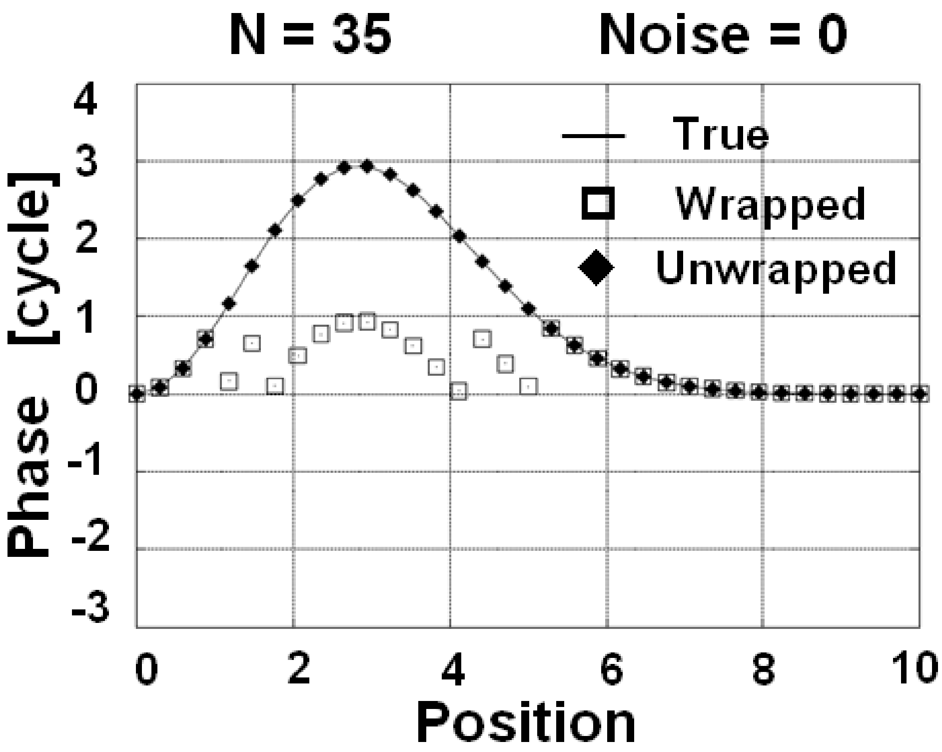
n: is integer number



Problems of phase unwrapping

After Extracting the phase we have only the wrapped phase.

If the extracted data are good (contain no error):

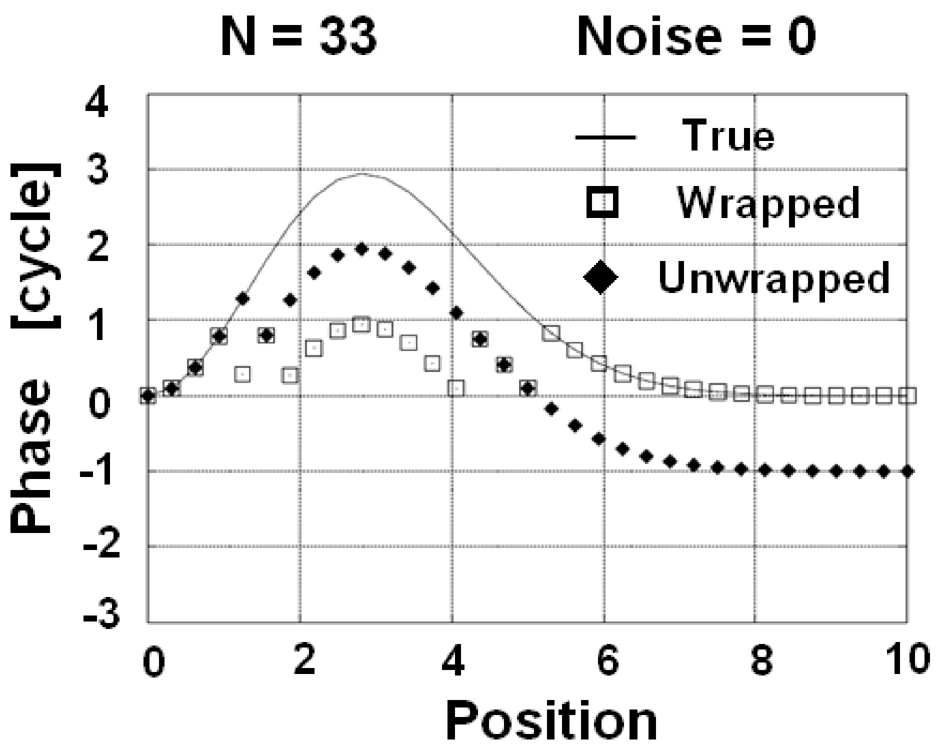


Phase unwrapping is straightforward

The unwrapped phase calculated by adding or subtracting an integer offset of 2π to the previous successive point.

Problems of phase unwrapping

➢ **Problems of phase unwrapping process:**



◆ **Violation of Shannon's theorem**:

(under-sampling in real wrapped phase maps)

Considering a boundary condition such as **the unwrapped phase must be zero at both ends**,

The unwrapping error includes at any point in the region can be found.

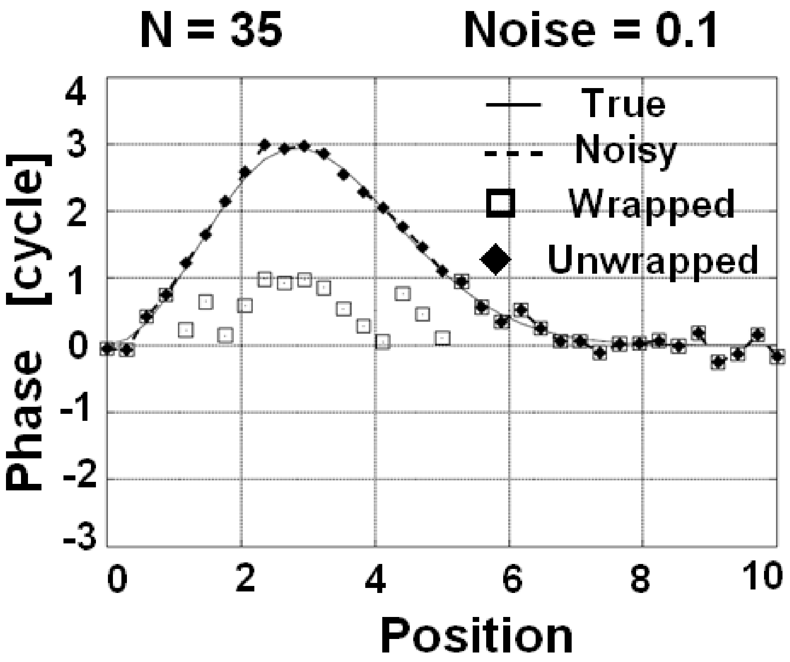
These additional conditions (boundary conditions) play an important role to fix the unwrapping errors

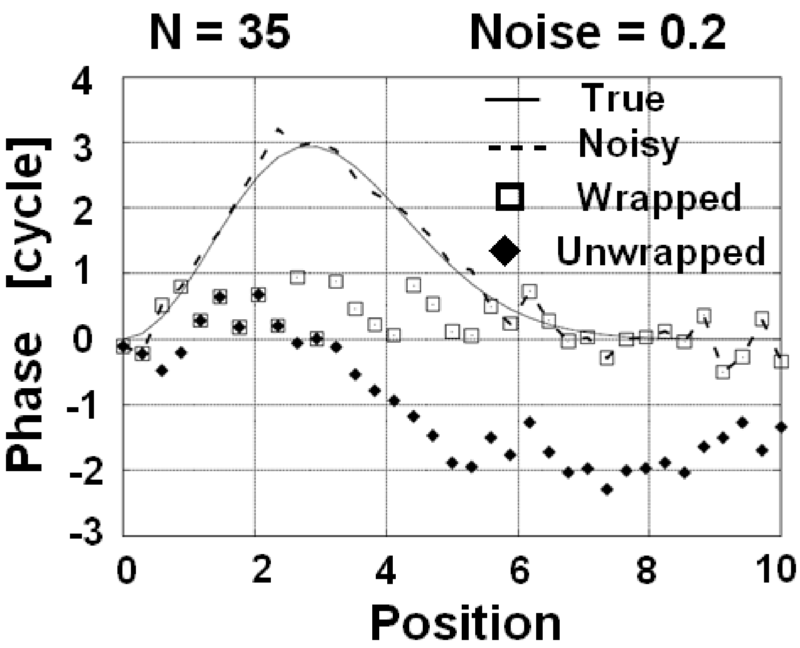
Problems of phase unwrapping

- **Problems of phase unwrapping process:**
  - **Low signal-to-noise ratio of fringes:**

    (caused by speckle noise, objects discontinuities and/or fringe breaks)

When **SNR is high** this is not consider big problem and unwrapping process still can be done successfully





Research Objectives

To propose efficient and robust phase unwrapping methods for wrapped noisy images obtained from various optical applications

1

Investigating carefully the **phase unwrapping process and problems**;

Providing a brief overview of the existing phase unwrapping methods and ideas;

2

Figure out the ideas and prove them to propose efficient unwrapping algorithms;

Implement the ideas to have applicable phase unwrapping algorithm;

examine the proposed methods on simulated and actual phase data;

3

Carry out comparisons between the proposed algorithms and the existing phase unwrapping methods ;

To evaluate the quality and validity for the unwrapped images taking in consideration the noise level and time cost for the simulated data

2

PHASE EXTRACTION AND UNWRAPPING PROCESSES

- Phase extraction and calculation
- Phase unwrapping
- Existing phase unwrapping algorithms
- Conclusions

Phase extraction and calculation

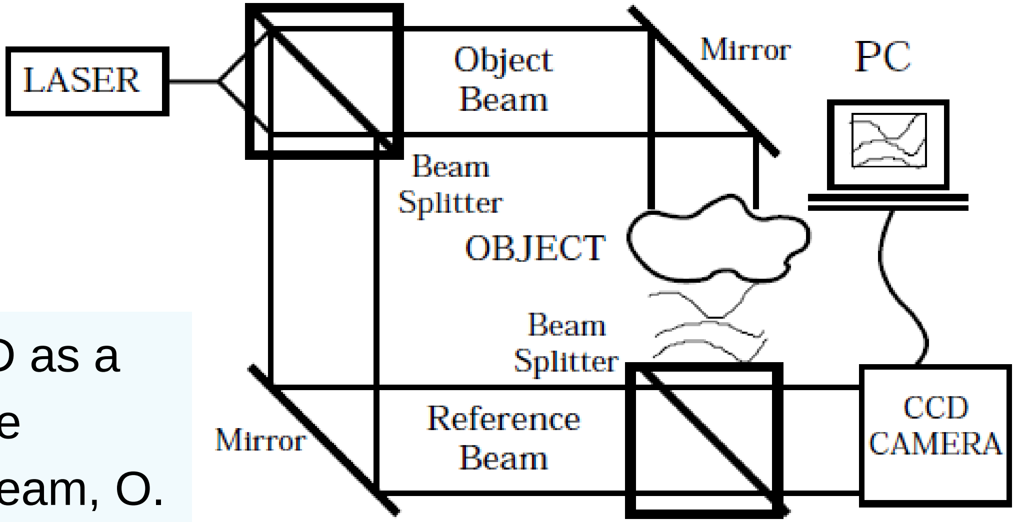
Interferometric measurement is considered as one example of application that is based on phase measurements to obtain its desired result.

Interferometer is a technique of measuring the phase modulation from the light reflected or transmitted from a projected object to screen in the form of interference pattern.

The optical setup of interferometric system

- **For the measuring**, light from a laser is divided by a beam splitter into two beams, one for object illumination and another for a reference.

- **Interferogram** is formed on the CCD as a result of the interference between the reference beam, R, and the object beam, O.



Interferometric measurements

The intensity recorded on the CCD is given:

$$I(\vec{r}) = I_0 + I_1 \cos\left(\underline{\vec{k}\cdot\vec{r} + \varphi}\right)$$

$$= I_0 + I_1 \left\{e^{i\left(\underline{\vec{k}\cdot\vec{r}+\varphi}\right)} + e^{-i\left(\underline{\vec{k}\cdot\vec{r}+\varphi}\right)}\right\}$$

Where: $I_0 = |O|^2 + |R|^2$ & $I_1 = 2|O||R|$

To retrieve the phase information, two techniques are partly used.

- One is the **phase shift interferometry** - several fringe patterns are recorded by varying the known phase shift introduced to one of the beams in the interferometer system.
- The other technique is spatial filtering for Interferogram using the **Fourier transform method** requires only one fringe pattern.

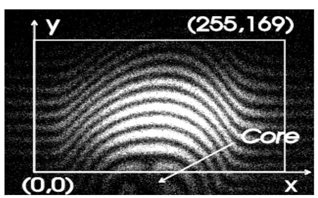
Interferometric measurements

The intensity recorded on the CCD is given:

$$I(\vec{r}) = I_0 + I_1 \left\{ e^{i\underline{\left(\vec{k}\cdot\vec{r}+\varphi\right)}} + e^{-i\underline{\left(\vec{k}\cdot\vec{r}+\varphi\right)}} \right\}$$

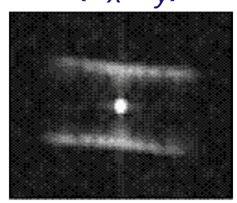
A schematic diagram describing the use of the Fourier transform method

$I(x,y)$



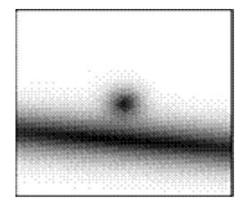
$\mathcal{F}$

$\hat{I}(k_x,k_y)$

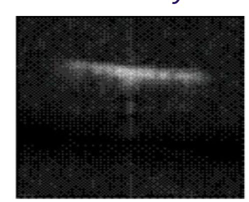


Filtering

$F(k_x,k_y)$

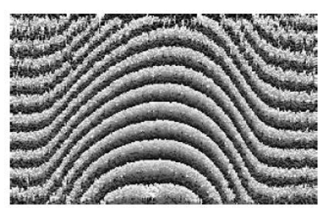


$\hat{I}_F(k_x,k_y)$



$\mathcal{F}^{-1}$

$W\{\varphi\ (x,y)\}$



$I(x,y)$: Interferometic data

$\hat{I}(k_x,k_y)$: Fourier spectrum

$F(k_x,k_y)$: Filtering function

$\hat{I}_F(k_x,k_y)$: Filtered spectrum

$W\{\varphi\ (x,y)\}$: Wrapped phase

Interferometric measurements

- After Fourier transformation and filtering to eliminate DC component and one of two side components:

$$\boxed{I'(\vec{r}) = I_1 e^{i\left(\vec{k}\cdot\vec{r}+\varphi\right)}}$$

- **From the complex amplitude**, the phase is calculated

$$\boxed{I''(\vec{r}) = I_1 e^{i\varphi}}$$

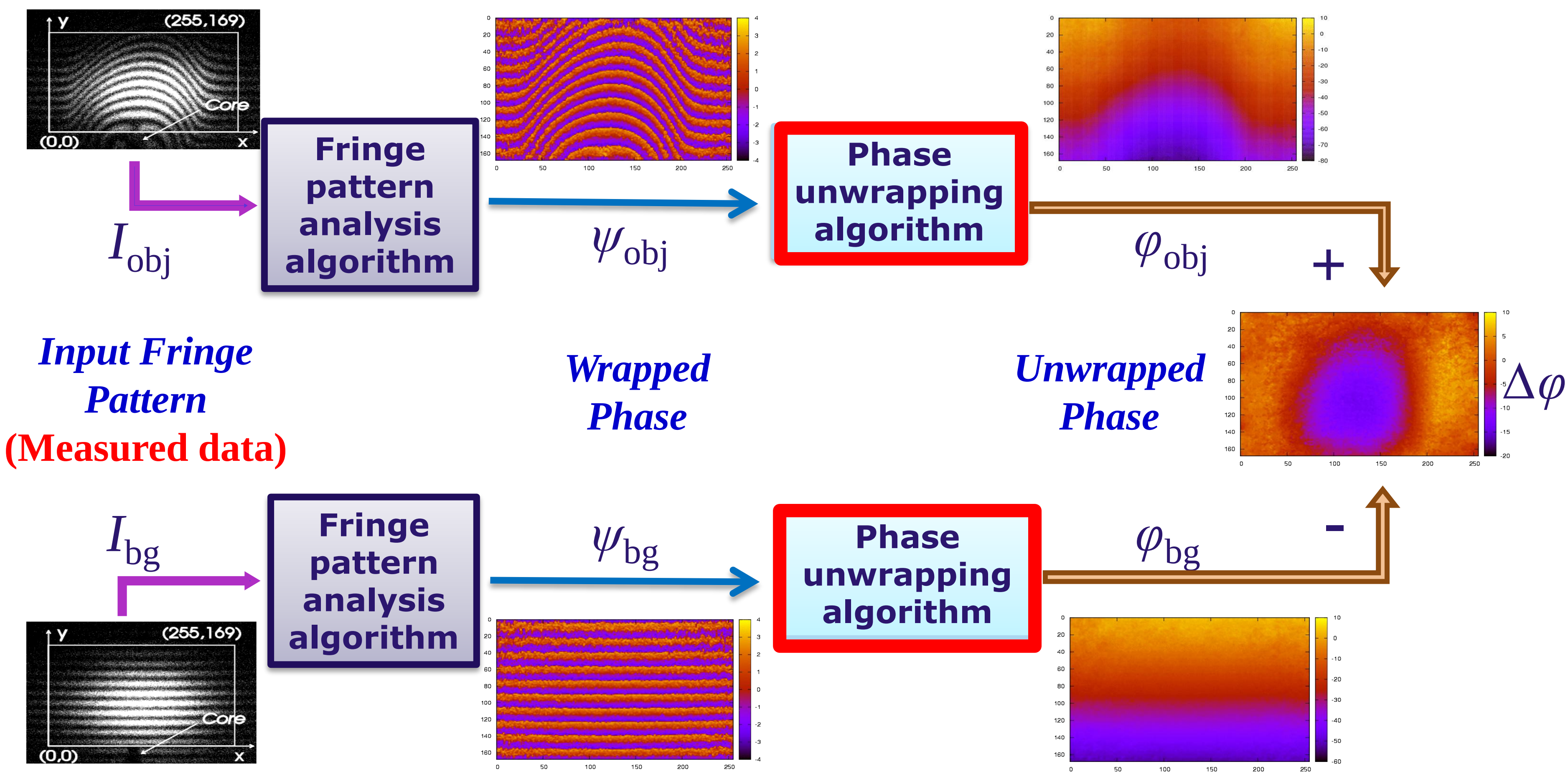
$$\downarrow$$

$$\boxed{\varphi(\vec{r}) = \frac{1}{i}\log\frac{I''(\vec{r})}{I_1}}$$

wrapped phase

Example of using phase unwrapping

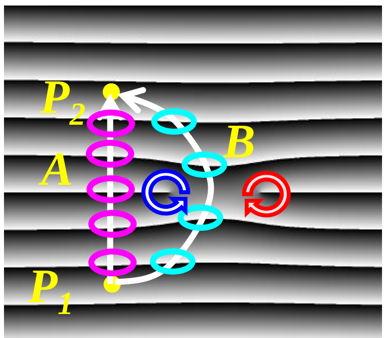
The target is temperature measurement of candle flame. By measuring the phase shift of the flame with optical interferometer

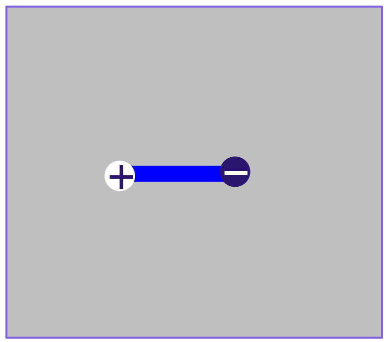


Singularity in phase unwrapping

Singularity ⇔ **Inconsistency** ⇔ **Singular Point (SP)**

Marks the beginning or end of the 2π discontinuity





SPs occurred due to:

- Noise during measurement
- Sampling problem
- Real phase gaps

Existing of SPs makes:

- Phase map inconsistent
- The unwrapping process path dependent

<u>Phase difference between P_1 and P_2</u>

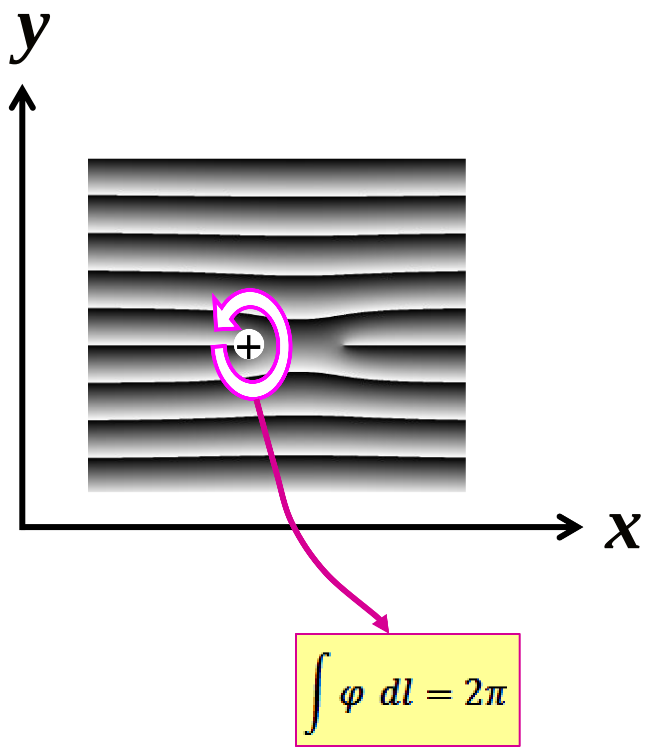
path A $\rightarrow \triangle\varphi$ is 10π rad

path B $\rightarrow \triangle\varphi$ is 8π rad

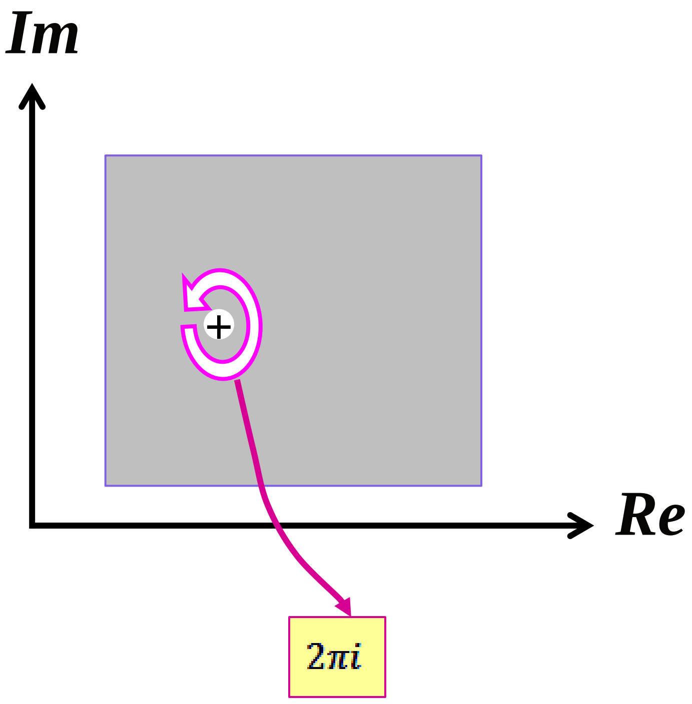
Existence of many SPs **make phase unwrapping problem more complicated**

Singularity in phase unwrapping

Phase Singularity



Complex Singularity



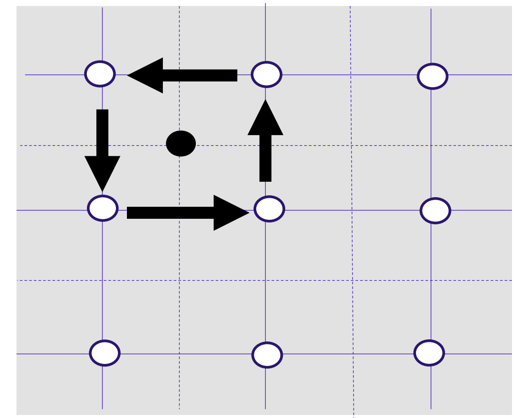
The singularity in the phase unwrapping has the same meaning to that in mathematical complex function analysis.

Singularity in phase unwrapping

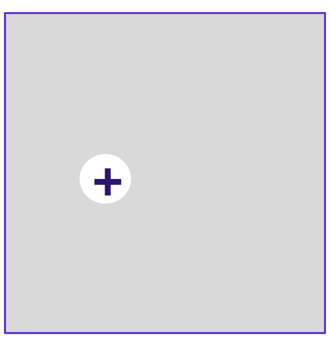
To find singular point **(SP)** in the phase map:

SPs are identified by summing the wrapped phase gradient, $\nabla\Psi_i$, as

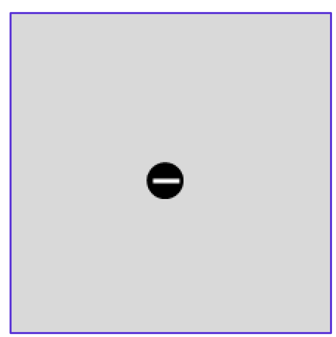
$$\sum_{i=1}^{N} \hat{\nabla}\Psi^{i} = 2\pi S$$



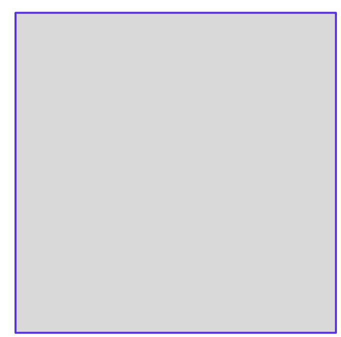
Consider a closed path starting in every point defined by the corners of a **2 × 2** square



SP is called a **positive residue** when $S = +1$



SP is called a **negative residue** when $S = -1$

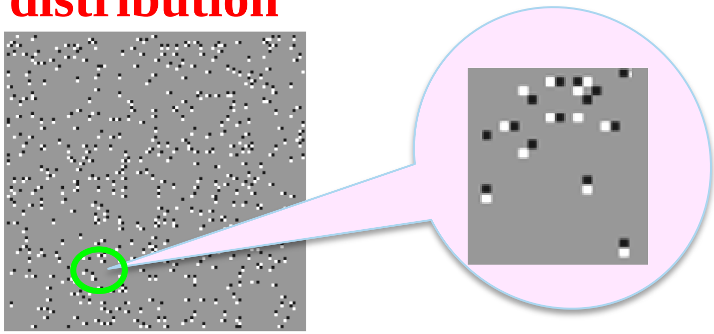


When $S = 0$ indicates that **no residue exists**

Regular region (Analytical)

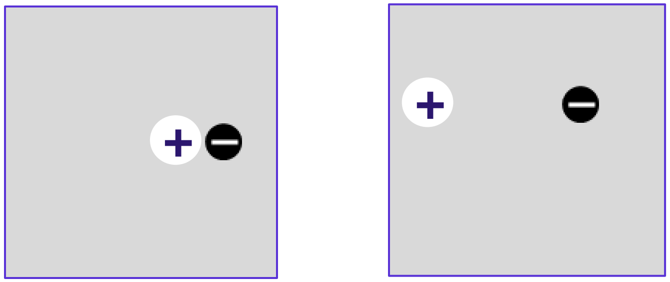
Singularity in phase unwrapping

SPs distribution

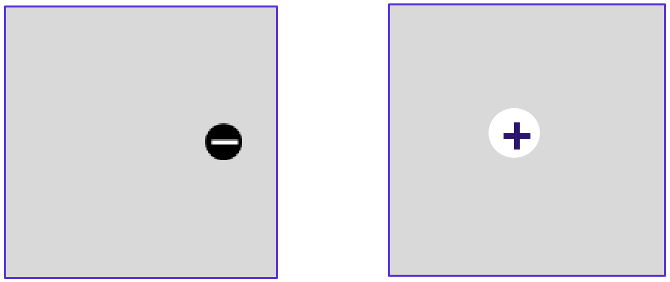


Dependent on the distribution of SPs in the phase map
There are two types

- **Dipole SPs**
  exist in pairs of two opposite polarity states



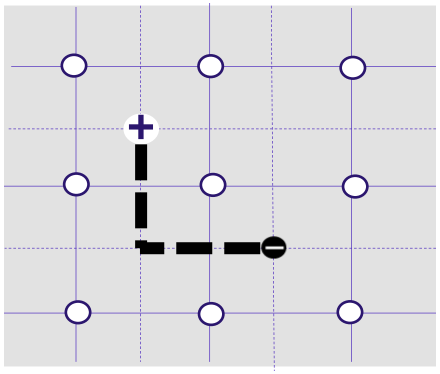
- **Monopole SPs**
  single value residues without corresponding opposite-sign partner

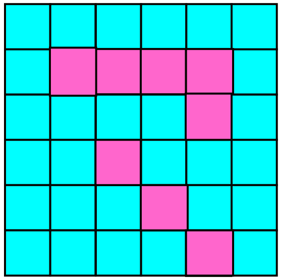


Branch cut in phase unwrapping

Branch cuts act as barriers to prevent the unwrapping path to cross them

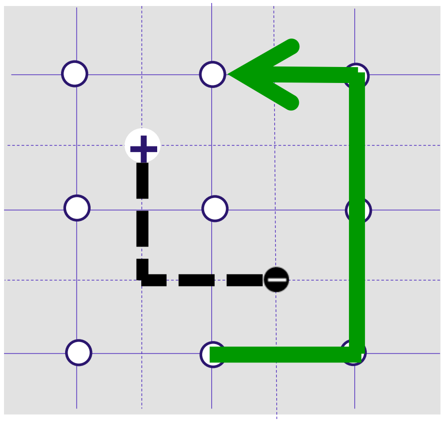
They connect between positive and negative SPs to avoid path-dependent results



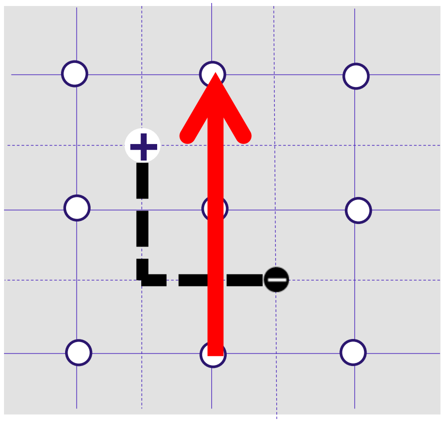


Branch cut in phase unwrapping

- If branch cuts are avoided during the unwrapping process
- no errors propagate and the unwrapping path is considered path independent



- If branch cuts are penetrated during the unwrapping process
- errors propagate throughout the whole phase map, and the unwrapping path is considered path dependent



Existing phase unwrapping algorithms

Path-following methods using branch cut

- They are a pixel-to-pixel integration techniques along a chosen path to construct the correct true phase.
- However, in the presence of noise or corrupted areas in the wrapped phase map the path of integration becomes dependent.
- **Examples:** Goldstein et al.'s path-following method (***Goldstein***)
  {Goldstein et al., J. Radio Science 4, p 713 (1988)}

  Flynn's minimum weighted discontinuity method (***Flynn***)
  {Flynn, J. Opt. Soc. Am. A 14, p 2692 (1997)}

Algorithms spread the effect of SP to the whole area of image

- They minimize up the difference between the gradients of the wrapped and unwrapped solution to a certain degree .
- They indirectly deal with the SP problem - their solution is obtained by integrating over the residues to minimize the gradient differences
- **Examples**: Least-Squares method with Discrete Cosine Transform (***LS-DCT***)
  {Ghiglia and Romero, J. Opt. Soc.Am. A 11, p107 (1994)}

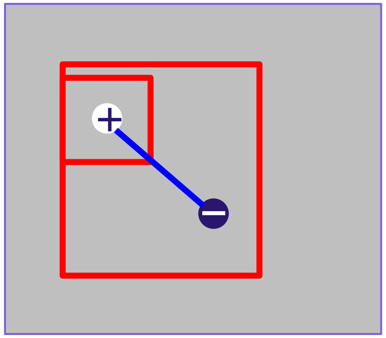
  Singularity-Spreading Phase Unwrapping method (***SSPU***)
  {R. Yamaki and A. Hirose, IEEE Trans. Geosci. and Remote Sen., p3240 (2007)}

Existing phase unwrapping algorithms

- **Goldstein et al.'s path-following method (Goldstein)**

1) Start with SP → 2) Search in box → 3) Find second SP → 4) Connect w branch cut

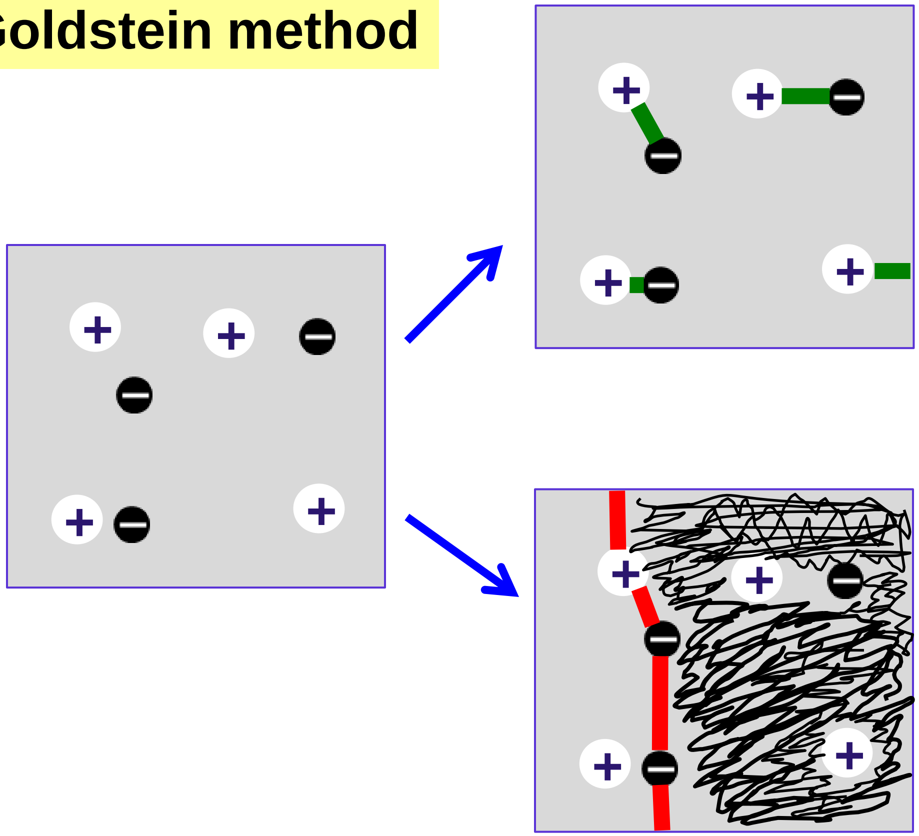
Minimizes the sum of the branch cut length



Existing phase unwrapping algorithms

Errors appear in the regular regions in the phase map due to the handling way of Goldstein method for singularity regions

Goldstein method



Correct placement of Branch cut

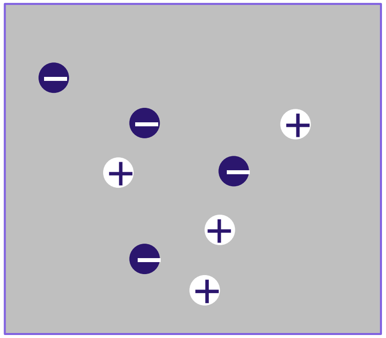
- Due to SNR is high so No. of SPs small
- Unwrapped results are perfect and does not have any errors

Incorrect placement of Branch cut

- Due to SNR is low so No. of SPs high
- Unwrapped results contain a lot of phase errors and jumps

◆ Flynn's minimum weighted discontinuity method (**Flynn**)

Common unwrapping strategy is to consider all discontinuities as defects and attempt to eliminate them



Finds a solution that actually minimizes the discontinuities

1) Partitioning the image into two connected sets of pixels

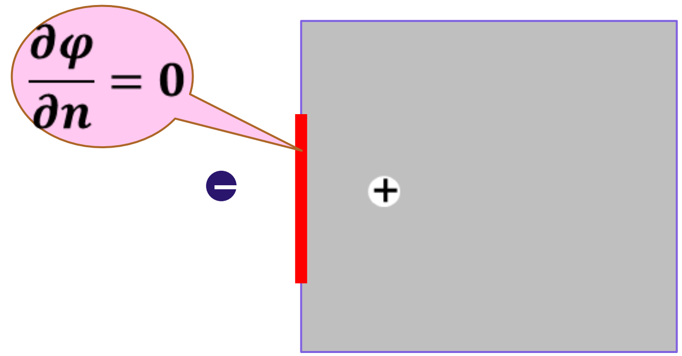
2) Increasing the unwrapped phase by 2π in one of the sets

A set of pixels is connected if any two pixels in the set can be joined by a sequence of horizontal or vertical neighbors that belong to the set

3) The set to be changed is the one that causes the weighted sum of discontinuities to decrease. The iterations stop when no partition is found.

Existing phase unwrapping algorithms

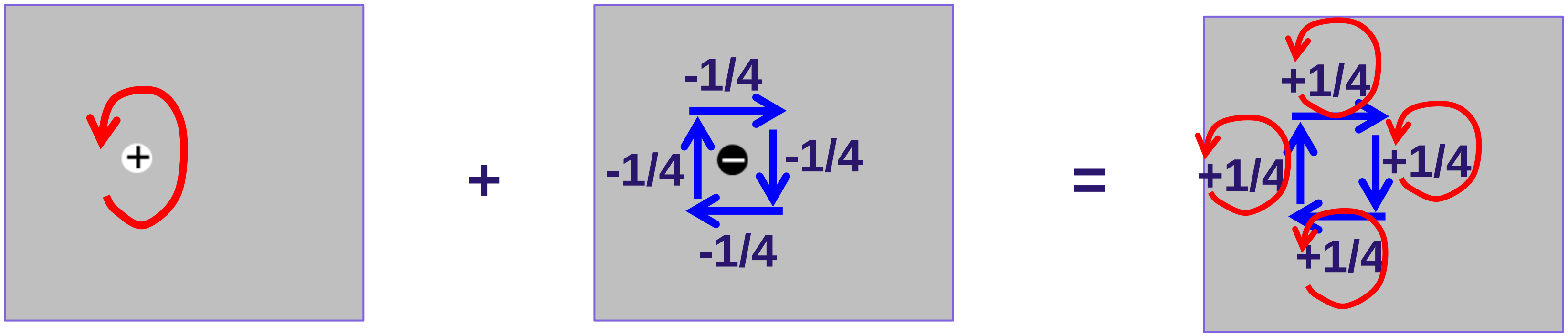
◆ **Least-Squares method with Discrete Cosine Transform (LS-DCT)**



Solve Poisson eq → $\vec{\nabla}^2 \varphi = \sum_i q_i$

under boundary condition → $\frac{\partial \varphi}{\partial n} = 0$

◆ **Singularity-Spreading Phase Unwrapping method (SSPU)**

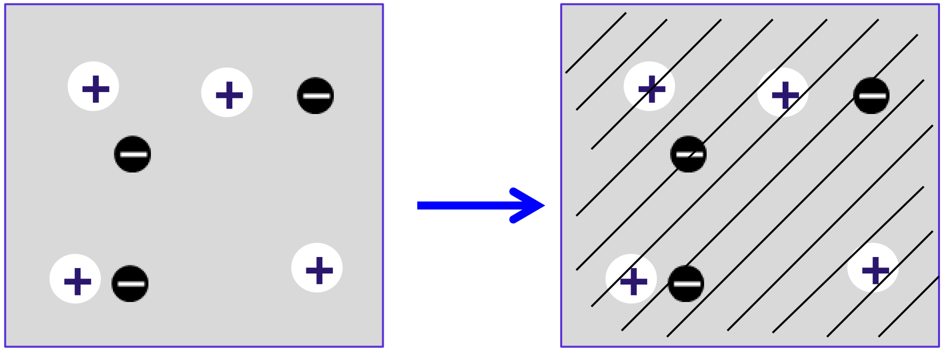


1) Defined SP → 2) Add compensators → 3) Spread Singularity

Existing phase unwrapping algorithms

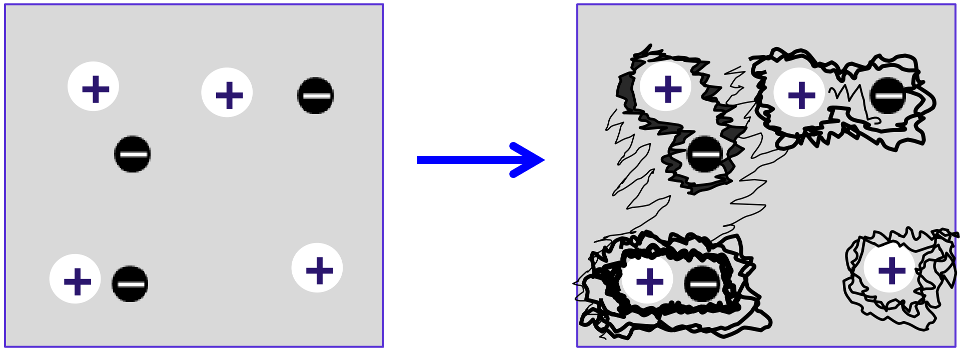
Errors appear in the regular regions in the phase map due to the way of handling of phase unwrapping algorithms for singularity regions

LS-DCT method



- **Spread the singularity to the whole domain**
- **Unwrapped result has error with unique density**

SSPU



- **Spread the singularity to the whole domain**
- **Phase error is decrease at regions far from SPs, but still big error**

Conclusions

1

A definition of the **phase unwrapping process** is presented, with a review of some of the **existing phase unwrapping algorithms**.

2

The problems that face phase unwrapping algorithms is described. The major problem for all phase unwrapping algorithms **is the SPs problem and their effects** on the unwrapping process.

3

There is clearly a need for **further investigation with particular emphasis to solve this problem** to produce acceptable results.

3 ROTATIONAL COMPENSATOR PHASE UNWRAPPING ALGORITHM

- Rotational compensator phase unwrapping method
- Results and discussion
- Conclusions

Rotational compensator phase unwrapping method (RC)

The developed algorithm is based on a combination of three approaches:

- Rotational Compensator (RC)
- Unconstrained Singular point Positioning (USP)
- Virtual Singular Points (VSP)

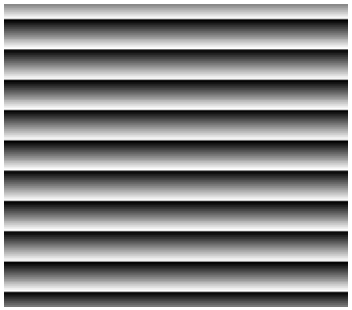
{ S. Tomioka, S. Heshmat, N. Miyamoto, and S. Nishiyama, J. Appl. Opt., p 4735 (2010) }

- **The RC method** evaluates the compensator directly without any iteration process.
- The accuracy is not much improved by only the RC technique itself.
- The other two USP and VSP are additional approaches to improve the accuracy.

- **USP approach** is used to confine the effect of compensator to smaller region and to determine the dipole pairs.
- **VSP** is located outside the measured area in which wrapped phase is obtained.

Rotational compensator technique

Wrapped phase distribution vertical gradient



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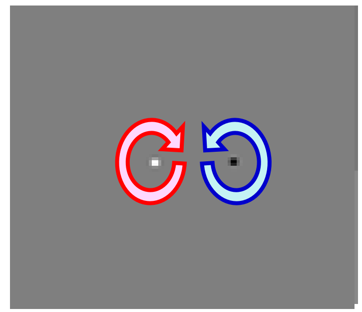
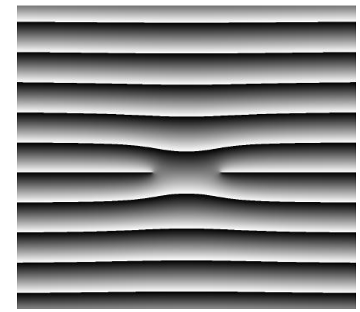
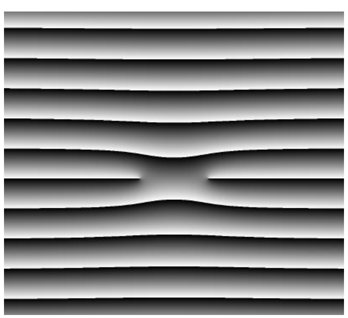
Phase map of two SPs



Measured data Wrapped phase contains SPs



Measured data contains SPs



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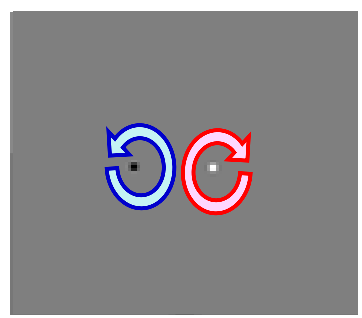
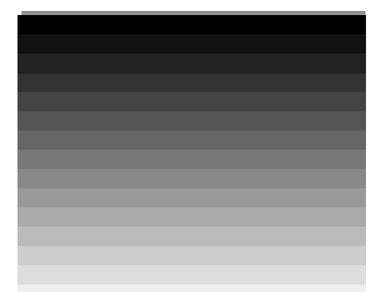
Rotational compensator

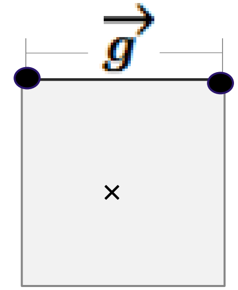


Regular phase distribution



Rotational compensator technique

If $\vec{g}$ is gradient of two adjacent points in elementary loop has SP



So

$$\oint \vec{g}\cdot d\vec{l} \neq 0$$

Satisfy

$$\vec{g} = \underbrace{\vec{\nabla}\varphi}_{\text{Phase}} + \underbrace{\vec{\nabla}\times\vec{A}}_{\text{Singularity}}$$

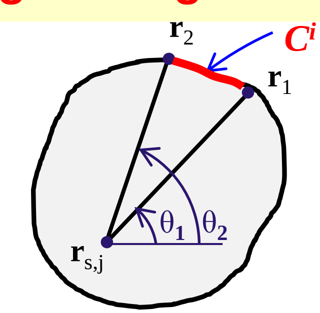
Helmholtz's theorem:
Any vector is represented by sum of two kinds of vectors **- irrotational vector and a rotational vector**

So → If $\vec{\nabla}\times\vec{A}$ is known

$$C_j^i = -\int_{r_1}^{r_2} (\vec{\nabla}\times\vec{A})\cdot dl = -m_j\,(\theta_2-\theta_1)\ \ \alpha\ \frac{dl}{R}$$

RC can **regularized the singularity,** **however** it **corrupts the other regular regions**

The phase difference between these adjacent points



$$\Delta\varphi = \int \vec{\nabla}\varphi\cdot d\vec{l} = \int (\vec{g}-\vec{\nabla}\times\vec{A})\,d\vec{l}$$

$$\boldsymbol{\varphi}(\boldsymbol{r}) = \boldsymbol{\varphi}(\boldsymbol{r_0}) + \Delta\varphi$$

VSP and USP techniques

The effect of single SP is considered as **monopole**

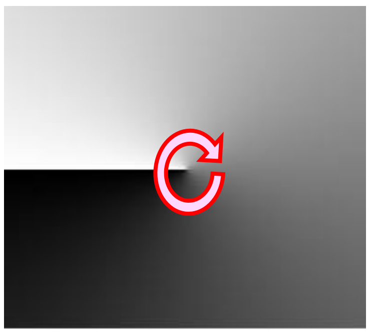
$$E_{\text{monopole}} = -\frac{1}{R} e_R.$$

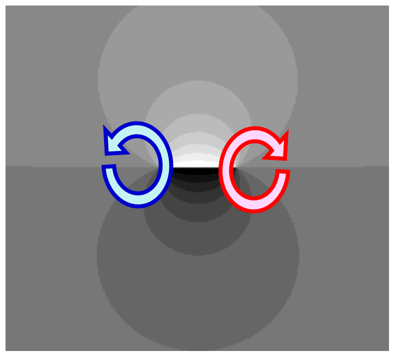
$\propto$ to the reciprocal of the distance R:

When dipole SP pair is found:

$$E_{\text{dipole}} = -\frac{1}{R^2} \left(2(\boldsymbol{d} \cdot \boldsymbol{e}_R)\boldsymbol{e}_R - \boldsymbol{d}\right)$$

where d is the difference vector from –ve SP to +ve SP



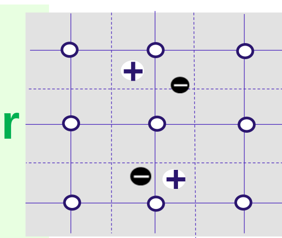


RC method **uses the VSP technique** to **coupling the monopole SP** near to the boarder to conform pairs

The decay of effect induced by a dipole is faster than that by a monopole.

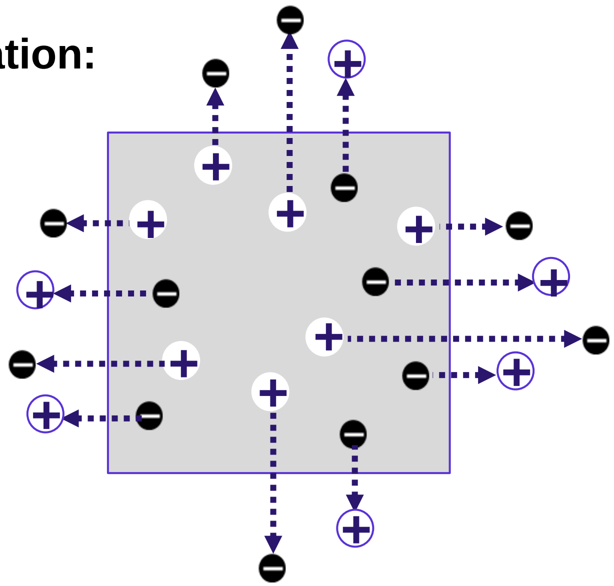
The E_{comp} is $|d|/R$ times smaller than that of a single compensator of monopole, so the phase error in the regular region will be reduced

When the accurate position of SP is obtained **by using USP technique** **the dipole distance is shorten than the pixel size** and **the compensator effect is limited to a narrower region.**

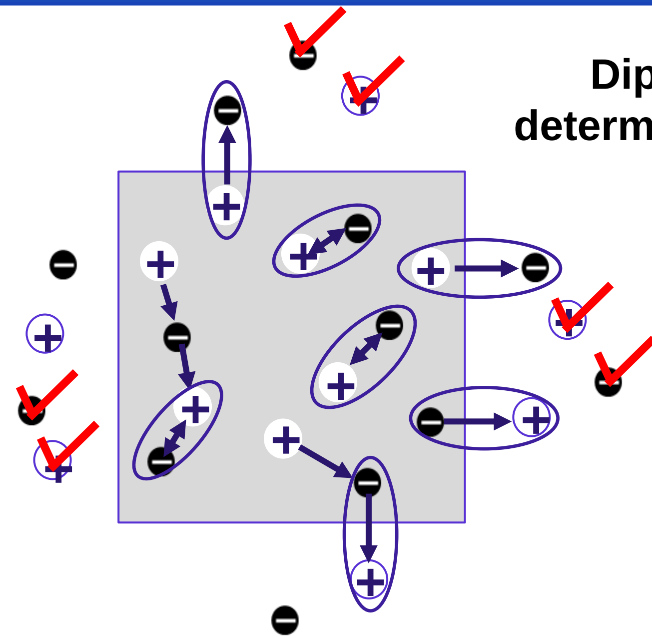


Virtual SP (VSP) technique

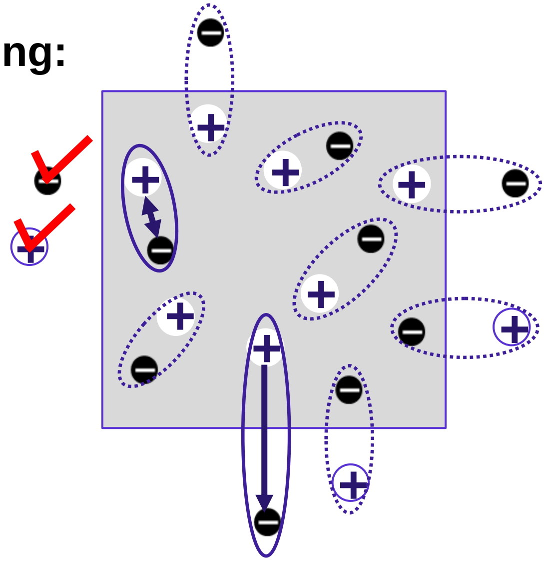
Preparation:



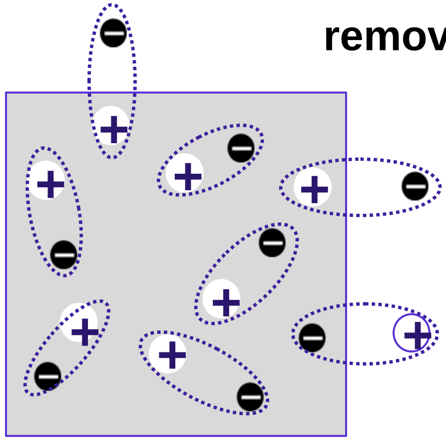
Dipole determination:



Repeating:



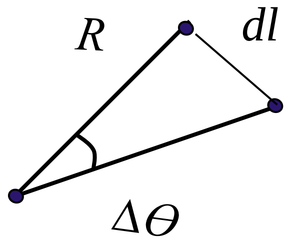
Elimination of removable VSP pair



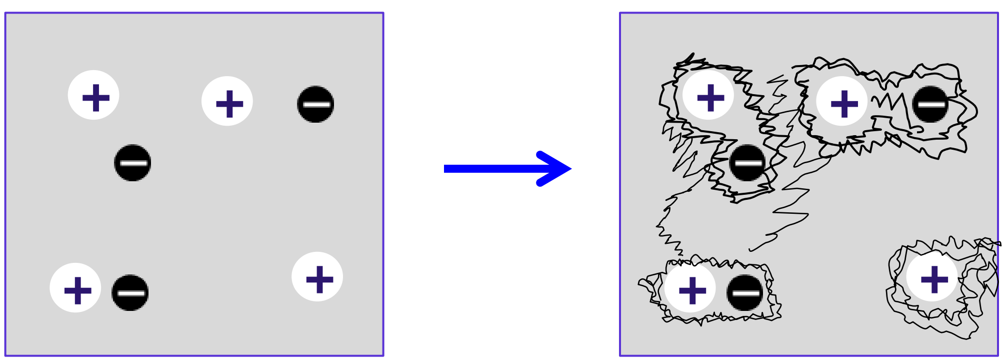
RC method

Errors appear in the regular regions due to the RC method handling way for singularity regions

$$C^k(\vec{r}_v,\vec{r}_2) = -m_k\Delta\theta \longrightarrow \alpha\,\frac{dl}{R}$$



∝ to the reciprocal of the distance *R*



- Spread the singularity as SSPU
- Phase error is decrease at regions far from SPs more than in SSPU , due to using of USP technique.

RC method Results (Simulation Data)

Original $\tilde{\varphi}$

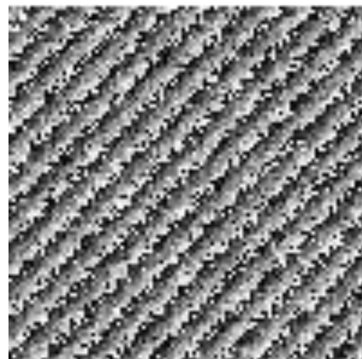
Required to estimate

(Unwrapped)

Measured data

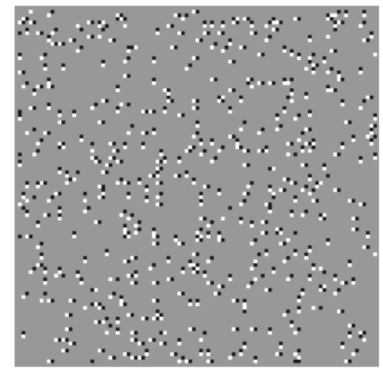
$(\psi) = W\{\tilde{\varphi}\}$

(Wrapped)



Residues map distribution

(SPs map)



- **The density of SPs is approximately 10%**
- 453 +ve residues and 456 –ve residues

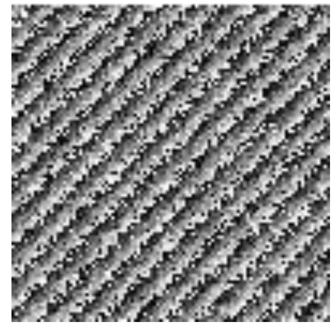
$$\tilde{\varphi}(x, y) = \bar{\varphi}'_x\, x + \bar{\varphi}'_y\, y + N(x, y)$$

$$(\bar{\varphi}'_x, \bar{\varphi}'_y) = (0.1\ , -0.1)\ [cycle/pixel]$$

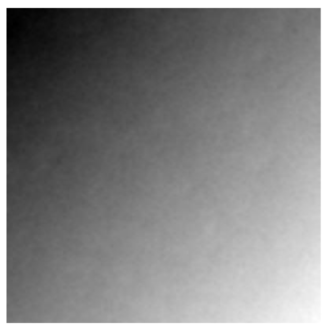
$$N(x, y) :\ normal\ distribution\ \left(\ \sigma: 0.15\ [cycle]\right)$$

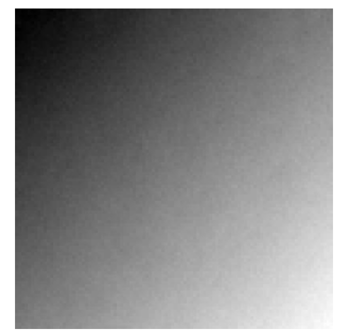
RC method Results

Original



Wrapped





Goldstein

LS-DCT

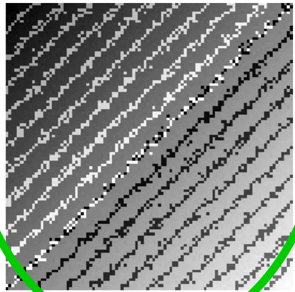
SSPU

RC

The old proposed

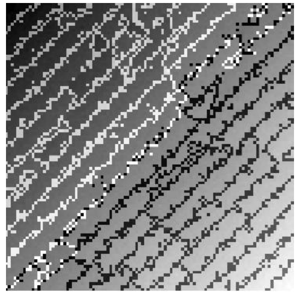
RC method Results

Original

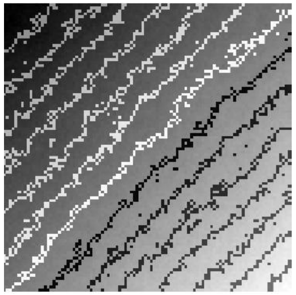


20 lines

Goldstein

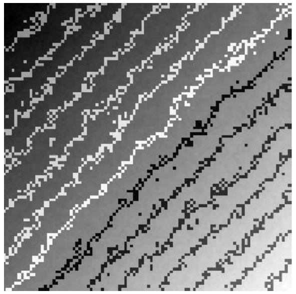


LS-DCT



14 lines

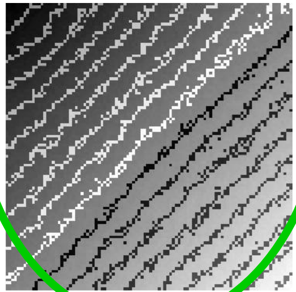
SSPU



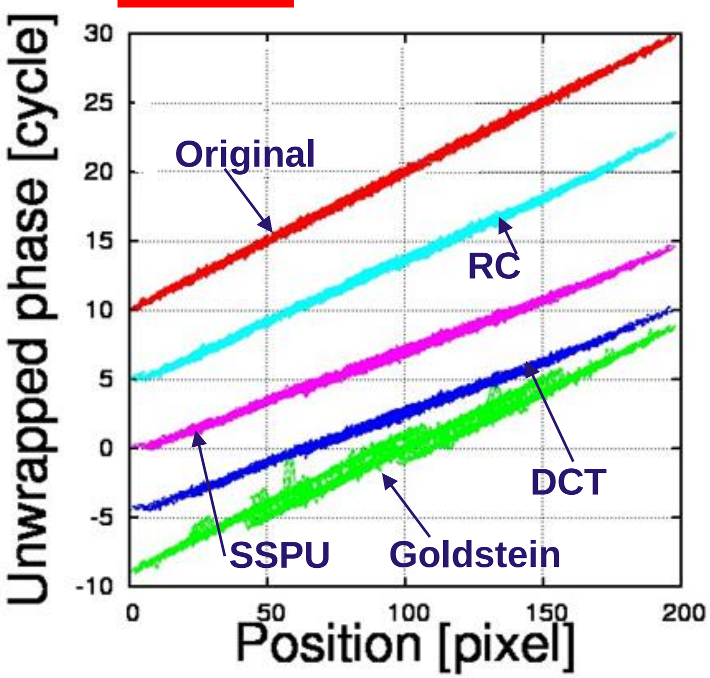
14 lines

RC

The old proposed



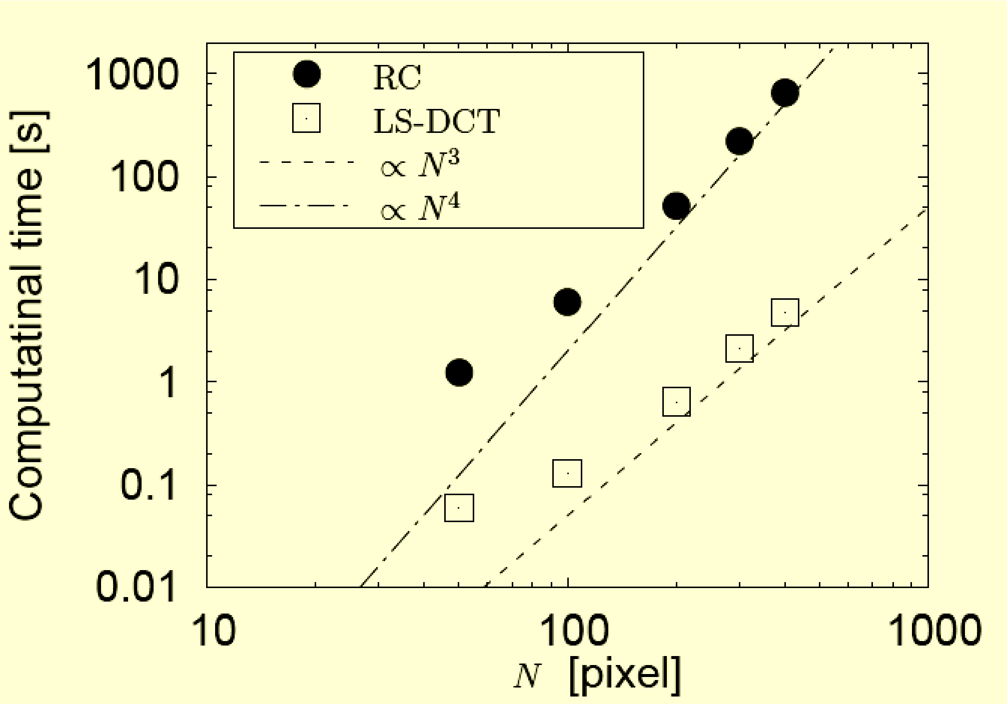
17 lines



Algorithm	Gradient $(\nabla\tilde{\phi})$	$\Delta\left(\nabla\tilde{\phi}\right)$ [%]	σ	$\Delta\sigma$ [%]
Original	(0.1000 , –0.1000)	(— , —)	0.149	—
Goldstein	(0.0892 , –0.0826)	(–10.8 , –17.4)	0.425	+184.9
LS-DCT	(0.0742 , –0.0731)	(–25.8 , –27.0)	0.179	+20.0
SSPU	(0.0743 , –0.0730)	(–25.7 , –27.0)	0.178	+19.7
RC	(0.0912 , –0.0896)	(–8.7 , –10.4)	0.168	+12.9

- **RC algorithm gives better correlation to the original data** compared to other algorithms, **due to the using of USP and VSP techniques**.
- but its result still has underestimation values

RC method Results



- Computation time required RC method is examined for different image sizes.

- LS-DCT method time cost increases with N^3,
- RC method time cost is almost $\propto$ to N^4; since it is $\propto$ to both the NO. of SPs and the NO. of the segments of path to be compensated
- **The RC method is more time-consuming than that of LS-DCT**

RC method Conclusion

1

We proposed RC algorithm, The RC can cancel singularity of each SP by adding an integral of isotropic singular function along any loops.

2

The RC algorithm had better accuracy for unwrapping noisy data among other methods, **but it still has underestimation result.**

3

There is another drawback for RC method, which is CPU cost.

4 ROTATIONAL AND DIRECT COMPENSATORS METHOD

- Direct compensator technique
- Phase unwrapping by rotational and direct compensators algorithm
- Results and discussion
- Conclusions

Rotational and direct compensators phase unwrapping method (RC+DC)

The drawbacks of RC method:

- **High cost of computational time**

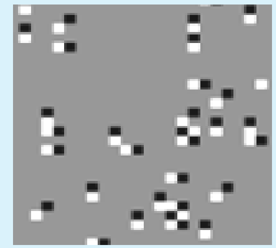
$$\mathbf{RC} \rightarrow C^{i} = \sum_{j}^{Ns} C_{j}^{i}$$

- **Undesired phase errors due to its effect on regular region,**

There is a necessity to propose a new phase unwrapping algorithm to estimate accurately phase maps with low computation cost.

Rotational and direct compensators phase unwrapping method (RC+DC)

- The distribution of dipole distance shows that there are a lot of SPs dipole pairs which have short distances:



An example of the distance distribution of the closest couple of SPs to each other.

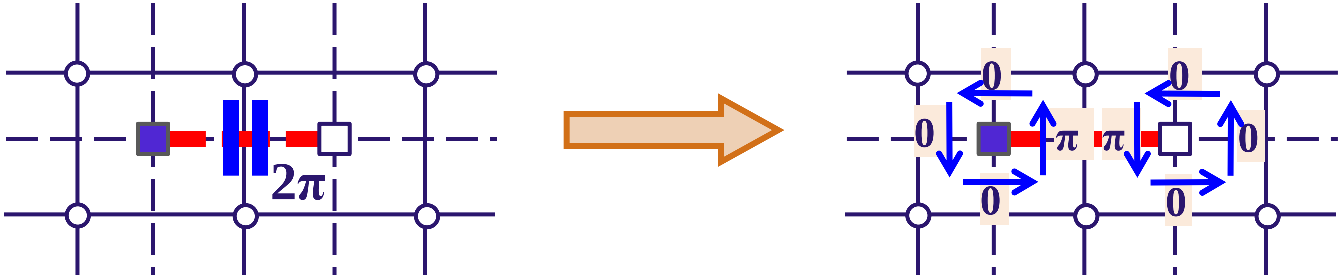
- The proposed algorithm is computing compensators for adjoining pairs of SPs directly, **hence computation cost will reduce**
- The new method is a coupling of the RC and the direct compensator.

Direct compensator technique

The proposed algorithm (RC+DC) offers simple computation to compensate the singularity effect of adjoining SPs pairs by adding a direct compensator (DC), As a result, the drawbacks of RC method will improve.

{ S. Heshmat, S. Tomioka, and S. Nishiyama, J. Appl. Opt. p **6225** (2011) }

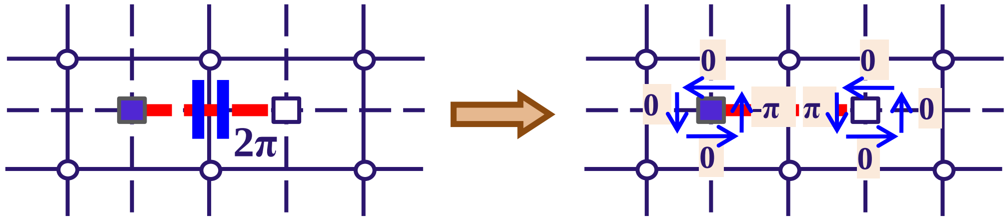
The adjoining pairs: is a dipole consist of two SPs with the opposite sign separated by one pixel horizontally or vertically



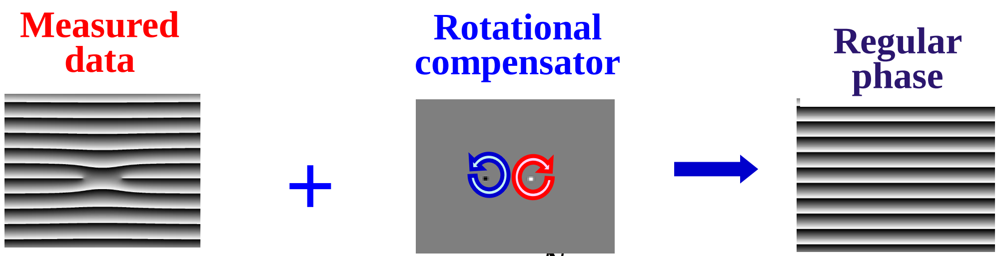
We compensate the singularity by adding the same value of SP with opposite polarity

If the SPs pair is adjoining pair:

- YES → A direct compensator will be added
  i.e. π radian (one half cycle)
  with opposite sign of each SP.



- NO → A rotational compensator will be computed



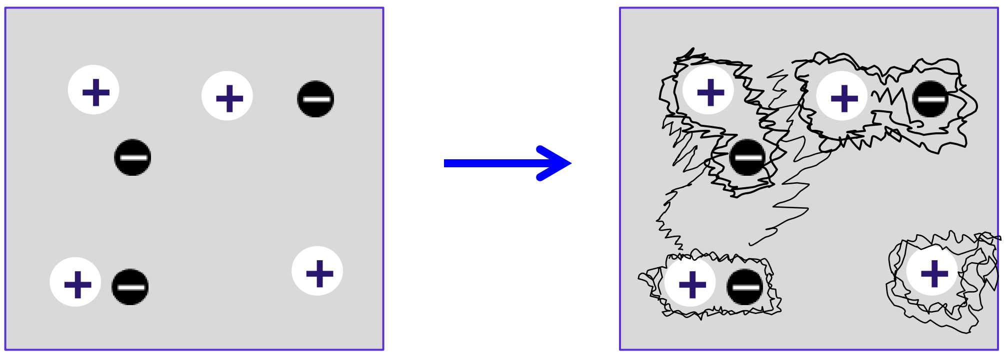
$$C^i = \sum^{Ns} C_j^i$$

This description of direct compensation for adjoining pair makes the proposed algorithm simple and easy to implement. It provides a fast and efficient way to unwrap the phase map.

Phase unwrapping compensation methods

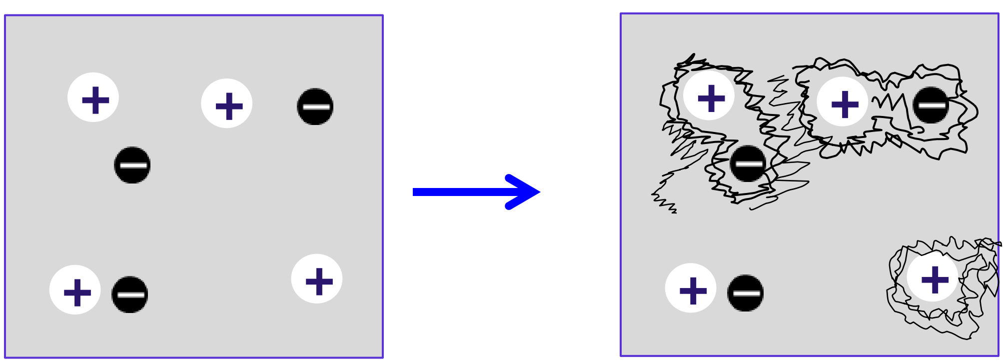
Errors appear in the regular regions in the phase map due to the way of handling of phase unwrapping algorithms for singularity regions

RC method



- Spread the singularity as SSPU
- Phase error is decrease at regions far from SPs more than in SSPU , due to using of USP technique, but still there is error

RC+DC method



- **Reduce the phase error for adjacent SPs to zero, by using DC technique.**
- But has errors due to using of RC for far distance of SPs dipole

RC+DC method Results (Simulation Data)

Original $\tilde{\varphi}$

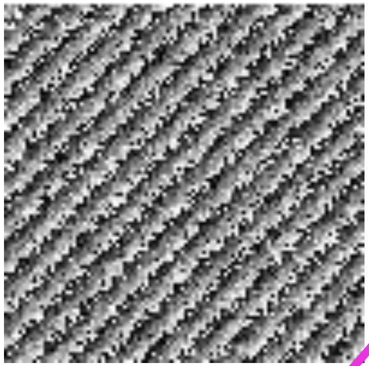
Required to estimate

(Unwrapped)

Measured data

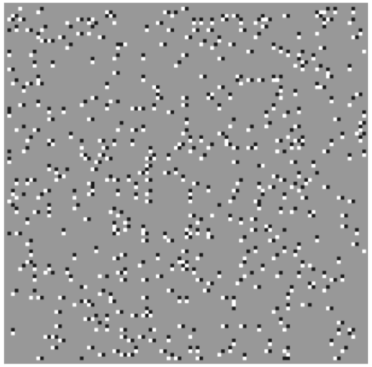
$(\psi) = W\{\tilde{\varphi}\}$

(Wrapped)



Residues map distribution

(SPs map)



- The density of SPs is approximately 10%
- 453 +ve residues and 456 –ve residues

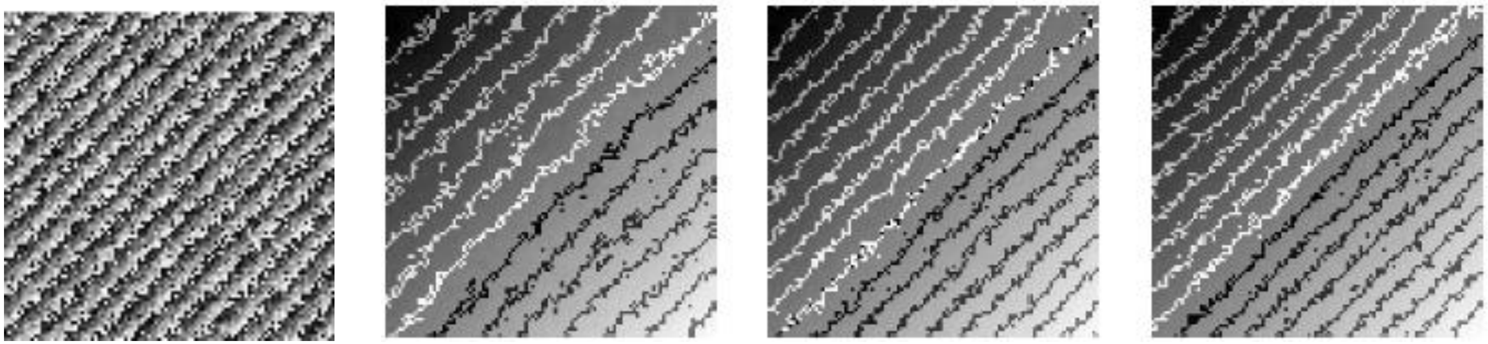
$$\tilde{\varphi}(x, y) = \bar{\varphi}'_x\, x + \bar{\varphi}'_y\, y + N(x, y)$$

$$(\bar{\varphi}'_x, \bar{\varphi}'_y) = (0.1\ , -0.1)\ [\mathit{cycle/pixel}]$$

$$N(x, y):\ \mathit{normal\ distribution}\ (\ \sigma{:}\ 0.15\ [\mathit{cycle}])$$

RC+DC method Results

Wrapped	LS-DCT	RC	The proposed (RC+DC)

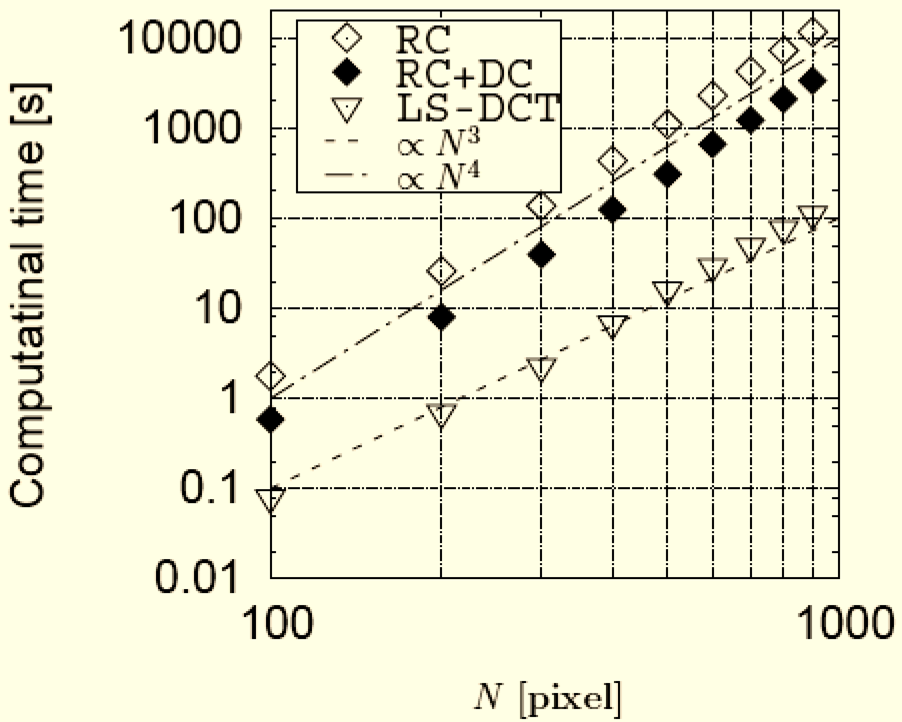


20 lines	14 lines	17 lines	18 lines

- ✓ The number of lines in the unwrapped results is less than that in the wrapped phase data.
- ✓ **The unwrapped result of the proposed algorithm has the nearest number of lines as wrapped data**

	Algorithm	Gradient $(\nabla\tilde{\phi})$ [cycle/pixel]	$\Delta\left(\nabla\tilde{\phi}\right)$ [%]	σ [cycle/pixel]
	Original	(0.1000 , –0.1000)	(— , —)	0.149
(a)	LS-DCT	(0.0742 , –0.0731)	(–25.8 , –27.0)	0.179
(b)	RC	(0.0912 , –0.0896)	(–8.7 , –10.4)	0.168
(c)	RC+DC	(0.0956 , –0.0951)	(–4.4 , –4.9)	0.168

RC+DC method Results



- Computation time required RC and RC+DC methods is examined for different image sizes.

Data name	Data size: N_{all}	SPs ratio: N_s/N_{all} [%]	Adjoining SPs ratio: N_a/N_s [%]	RC T_{total} [s] T_{ovrh} [s] T_{comp} [s]	(RC+DC) T_{total} [s] T_{ovrh} [s] T_{comp} [s]	Saving time ratio [%]
Noisy phase	100x100	9.1	80.9	1.781	0.585	67.2
				0.240	0.240	
				1.541	0.345	77.6

- The execution time to search and analyze SPs is the same for RC and RC+DC methods
- The execution time to compute the compensators in the RC+DC is reduced due to use of DC for the adjoining pair SPs.

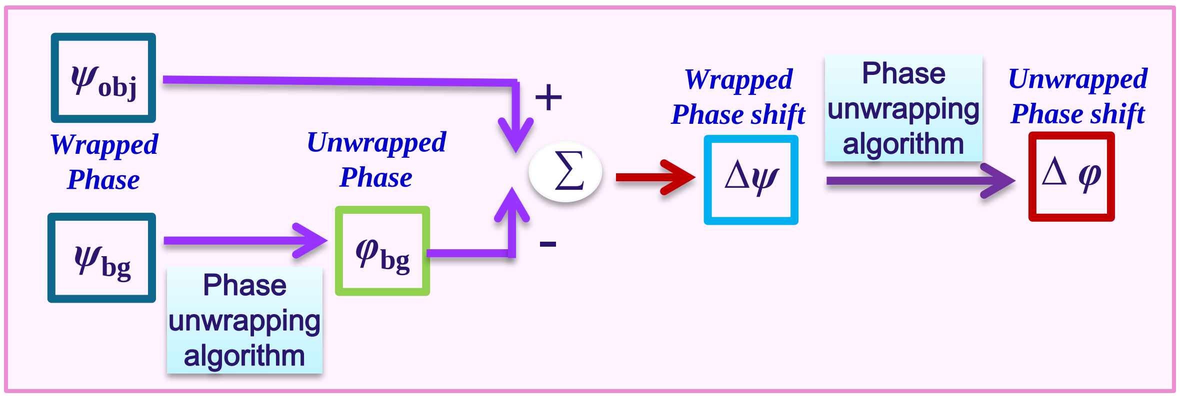
RC+DC method Results (Interferogram)

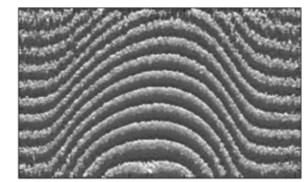
In the interferometric measurement system, two fringe patterns should be measured to produce information of an object

1. Object fringe pattern (obj)
2. Reference fringe (bg)

There are two ways to extract the phase shift caused by the object from the measured data

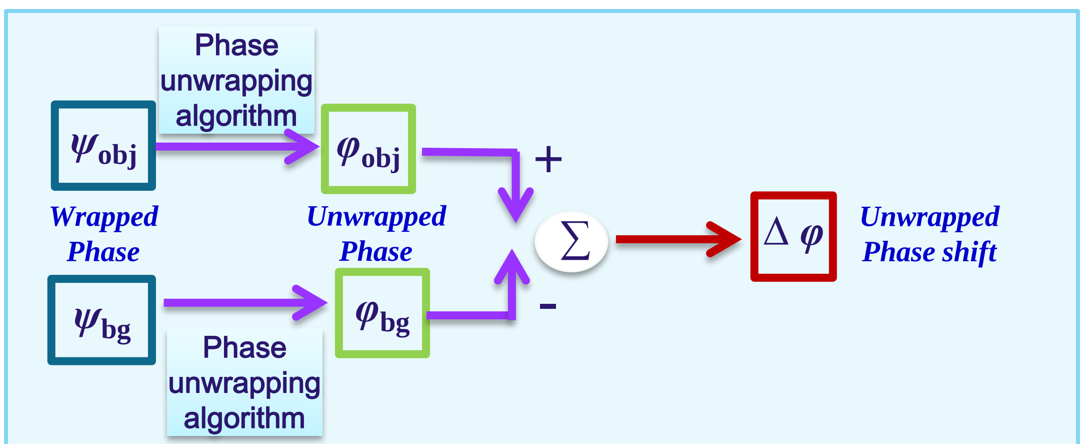
Pre-Rejection BG

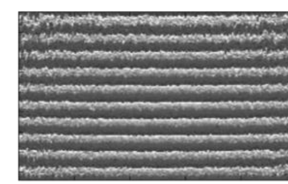




ψ_{obj}

Post-Rejection BG

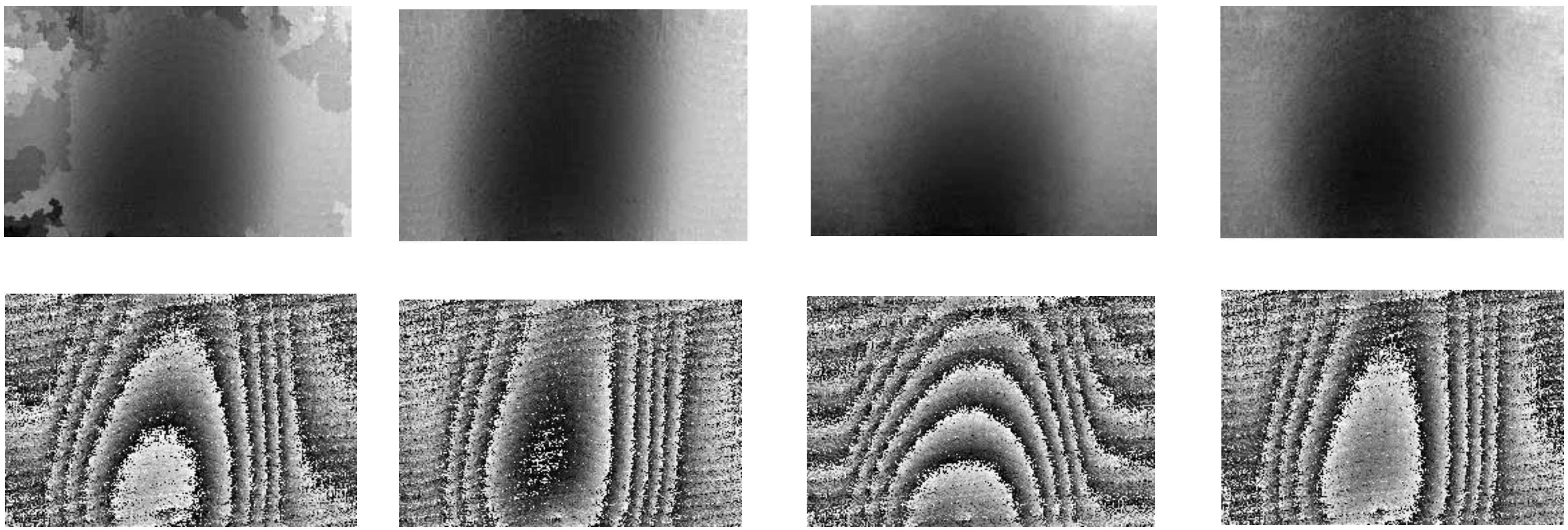




ψbg

RC+DC method Results (Interferogram)

The object is the temperature measurement of the heated gas (Air) around a candle flame through measuring the phase shift caused by the flame using Mach-Zehnder interferometer



Unwrapped

Rewrapped

Goldstein

Flynn

736.60 sec

LS-DCT

The proposed (RC+DC)

8.84 sec

- Goldstein causes phase jumps
- Other methods have no phase jumps
- The shape of LS-DCT rewrapped result is not looks like flame shape
- **Flynn and RC+DC provide accurate results**
- **Time cost of Flynn is too high compare with RC+DC**

RC+DC method Conclusion

1. RC algorithm had good accuracy for unwrapping noisy data among other methods. However it has drawbacks of CPU cost and phase error due to its effect on regular regions.

2. We proposed new method coupled rotational and direct compensation to remove the effect of singularities (RC+DC) method.

3. The proposed method (RC+DC) gives results with acceptable quality and with low computation time.

5 CONCLUSIONS

Conclusion

1

The dissertation has made investigations in fringe pattern analysis process, **specially for phase unwrapping stage**. A general review to the main stages for fringe analysis process has been introduced.

2

The phase unwrapping problem was presented, and the problems that face many phase unwrapping algorithms have been briefly described. **The major problem for all phase unwrapping algorithms is the singularity problem.**

3

This research has presented two novel phase unwrapping methods **for the purpose of more accurate phase unwrapping for noisy wrapped phase maps** for various optical applications. area.

4

The proposed algorithms were tested on both simulated and experimental phase data. In regard to the unwrapped phase results of simulated data, the proposed algorithms give the best solution with high quality compared to the examined algorithms .

Thank You !

Samia HESHMAT

samia@athena.qe.eng.hokudai.ac.jp